where (6) is the well-known cubic refractive index equation [3] for the longitudinally drifting magnetoplasma, with respect to the circular frequency ω_T of the transmitted plasma waves. From (3) and by dividing (5) one may obtain the wave polarization of the plasma waves

$$R = \frac{E_y}{E_x} = \frac{P_y}{P_x} = -\frac{H_x}{H_y} = \mp i \qquad (7)$$

where (7) is identical with the classical magneto-ionic theory for stationary plasma [4].

The effective values of the electric field $\bar{E}_{eff}$ and magnetic field $\bar{H}_{eff}$ of moving medium are given [5] by

$$\bar{E}_{eff} = \bar{E} + \bar{v}_0 \times \mu_0 \bar{H} \qquad (8a)$$

$$\bar{H}_{eff} = \bar{H} - \bar{v}_0 \times \epsilon_0 \bar{E}. \qquad (8b)$$

It has been shown [6] that the tangential boundary conditions at a moving boundary require

$$\bar{n} \times [\bar{E}_{eff}]_-^+ = 0, \quad \bar{n} \times [\bar{H}_{eff}]_-^+ = 0 \qquad (9)$$

where there is no surface current density on the boundary and $[F]_-^+$ represents discontinuity $F^+ - F^-$ with $\pm$ signs relative to $\bar{n}$. Substituting (8) into (9) for the present case of no $\hat{z}$ field components, one has

$$[\hat{z} \times \bar{E} - v_0\mu_0\bar{H}]_{z=v_0t-} = [\hat{z} \times \bar{E} - v_0\mu_0\bar{H}]_{z=vt_0+} \qquad (10a)$$

$$[\hat{z} \times \bar{H} + v_0\epsilon_0\bar{E}]_{z=v_0t-} = [\hat{z} \times \bar{H} + v_0\epsilon_0\bar{E}]_{z=vt_0+} \qquad (10b)$$

where $z = v_0t-$ represents the free space side and $z = v_0t+$ represents the plasma side of the drifting boundary.

In order to obey the boundary conditions (10) at the drifting boundary, all waves will be required to have the same exponential time variation at $z = v_0t$, i.e.,

$$\omega_I t - k_I v_0 t = \omega_R t + k_R v_0 t = \omega_T t - k_T v_0 t. \qquad (11a)$$

Using the above definitions one has the requirement

$$\omega_I(1 - \beta_L) = \omega_R(1 + \beta_L) = \omega_T(1 - n\beta_L) \qquad (11b)$$

where $\beta_L = v_0/c$, and $n = c/u_p$. Assuming that the source frequency ω_I of the incident wave is given, one may find

$$\omega_R = \omega_I \frac{1 - \beta_L}{1 + \beta_L};$$

$$\omega_T = \omega_I \frac{1 - \beta_L}{1 - n\beta_L}. \qquad (11c)$$

Because of the boundary conditions at the drifting boundary, the frequency of the reflected wave ω_R and of the transmitted wave in the plasma ω_T are given in terms of the frequency of the incident wave by (11c). Defining a new set of plasma parameters with respect to the circular frequency ω_I of the incident wave, denoting

$$X_I = \frac{\omega_p^2}{\omega_I^2}, \qquad Y_I = \frac{\omega_{HL}}{\omega_I},$$

and

$$Z_I = \frac{\nu}{\omega_I}$$

and using together with (11c) in (6), one obtains

$$(1 - \beta_L)(n^2 - 1)[1 - \beta_L - iZ_I \mp Y_I] + X_I(1 - n\beta_L)^2 = 0. \qquad (12)$$

Equation (12) is a quadratic equation for the refractive index $n = c/u_p$ of the drifting plasma, as compared to (6) which is a cubic equation. In general, one will have four different solutions for n in (12), two for $-Y_I$ and two for $+Y_I$; one of each pair will represent transmitted characteristic waves in the drifting plasma in the positive z direction and their refractive indices will be denoted by n_1 and n_2, respectively.

Using (3a) and (7) one has for the first transmitted plasma wave ($n = n_1$; $R = -i$)

$$\bar{E}_{T1} = E_x^{(1)}(\hat{x} - i\hat{y})e^{i(\omega_{T1}t - k_{T1}z)} \qquad (13a)$$

$$\bar{H}_{T1} = i\frac{n_1}{\eta}E_x^{(1)}(\hat{x} - i\hat{y})e^{i(\omega_{T1}t - k_{T1}z)} \qquad (13b)$$

where $E_x^{(1)}$ is a constant,

$$\omega_{T1} = \omega_I \frac{1 - \beta_L}{1 - n_1\beta_L}$$

and

$$k_{T1} = \frac{n_1\omega_{T1}}{c} = \frac{n_1\omega_I}{c}\frac{1 - \beta_L}{1 - n_1\beta_L}.$$

The corresponding second transmitted plasma wave ($n = n_2$, $R_2 = +i$) will be

$$\bar{E}_{T2} = E_x^{(2)}(\hat{x} + i\hat{y})e^{i(\omega_{T2}t - k_{T2}z)} \qquad (14a)$$

$$\bar{H}_{T2} = -i\frac{n_2}{\eta}E_x^{(2)}(\hat{x} + i\hat{y})e^{i(\omega_{T2}t - k_{T2}z)} \qquad (14b)$$

where $E_x^{(2)}$ is a constant,

$$\omega_{T2} = \omega_I \frac{1 - \beta_L}{1 - n_2\beta_L}$$

and

$$k_{T2} = \frac{n_2\omega_{T2}}{c} = \frac{n_2\omega_I}{c}\frac{1 - \beta_L}{1 - n_2\beta_L}.$$

Both waves are circularly polarized but in opposite directions; because of the boundary conditions at the drifting boundary, the waves will propagate with different circular frequencies ω_{T1}, ω_{T2} in the plasma for the same frequency ω_I of the incident wave.

Assuming that the amplitude E_x^I of the linearly polarized incident plane wave in (1) is known, one can find E_x^R, E_y^R of the reflected waves in (2) and $E_x^{(1)}$, $E_x^{(2)}$ of the transmitted waves in (13) and (14) by using the four boundary conditions in (10) at the drifting boundary $z = v_0t$. Substituting (1), (2), (13), and (14) in the boundary conditions (10), cancelling the identical exponential time variations at $z = v_0t$, taking

$$v_0\epsilon_0\eta = \frac{v_0\mu_0}{\eta} = \frac{v_0}{c} = \beta_L$$

and rearranging, one obtains

$$\begin{bmatrix} -(1+\beta_L) & 0 & +(1-n_1\beta_L) & +(1-n_2\beta_L) \\ 0 & -i(1+\beta_L) & +(1-n_1\beta_L) & -(1-n_2\beta_L) \\ 0 & +i(1+\beta_L) & +(n_1-\beta_L) & -(n_2-\beta_L) \\ +(1+\beta_L) & 0 & +(n_1-\beta_L) & +(n_2-\beta_L) \end{bmatrix} \begin{bmatrix} E_x^R \\ E_y^R \\ E_x^{(1)} \\ E_x^{(2)} \end{bmatrix} = \begin{bmatrix} 1-\beta_L \\ 0 \\ 0 \\ 1-\beta_L \end{bmatrix} E_x^I \qquad (15)$$

where (15) is identical with the result found by Chawla and Unz [1] for the relativistic case. Once n_1 and n_2 for the transmitted plasma waves are found from (12), one may find reflected waves E_x^R, E_y^R and the drifting plasma transmitted waves $E_x^{(1)}$, $E_x^{(2)}$ in terms of the incident wave E_x^I after multiplying (15) by the inverse of the square matrix.

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An Experimental Gas Lens Optical Transmission Line

The advent of successful gas lenses [1]–[4] started investigations of their performance in optical transmission lines. A test line of spaced thermal gas lenses was built at Holmdel in a long corridor using parts and techniques available from previous circular waveguide work.

The gas lens elements of this line consisted of copper tubes 15 cm long with an inside diameter of 6.3 mm and an outside diameter of 9.5 mm. They were heated by a single layer winding of number 34 enameled resistance wire having a total resistance of 1400 ohms. A thermocouple was soldered to the center of each lens tube to determine its temperature.

Each lens element was foamed in place in the center of a one-half meter length of 2-inch

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diameter circular copper tubing having the same precision alignment flanges used for TE_{01} waveguide. The polyurethane foam served as both mechanical support and heat insulation. The lens sections were assembled by fitting tapered mandrels into each end of a lens element to hold it in place in the 2-inch diameter tube during foaming. The mandrels served as forms for tapered foam passages from the small lens diameter to the 2-inch spacer diameter in order to control turbulence in the air flow. These mandrels were removed after curing the foam.

A section of the same 2-inch diameter copper tubing one-half meter long, identical with the sections used to hold the lenses, except empty inside, was mounted between each lens section, giving a one meter lens spacing. This spacer section cools the gas coming out of each lens before it enters the next lens, as a single gas feed is used at one end of the line, and the gas flows through the lenses from one end to the other in series. Air is normally used as the operating gas, although other gases can be used. Flow rate is usually in the order of $1\frac{1}{2}$ liters per minute.

In order to use such a transmission line consisting of a long series of these lenses, matching the input beam to the normal mode of the line is necessary. The method chosen here was to use a section of the line itself to generate its own normal mode. At mid-points between lenses the equiphase wavefront of this mode is plane. If a plane mirror is located at this position, the reflected wave sees the same transmission line characteristics that it would have if it had continued on in the initial direction through an identical lens line. If we place plane mirrors facing each other at this position with one or more lenses in the line between them, a Fabry-Perot cavity is formed, with its beam characteristics determined by the lens parameters and spacing. The addition of a gas laser tube in any section between lenses then makes an operating laser if the system loss is less than the laser tube gain. In the experimental line a helium-neon laser tube 35 cm long, operating at 6328 Å, with a discharge length of about 30 cm was placed in the space between the first and second lenses from the plane mirror mounted at the end of the line half the lens spacing before the first lens. A second plane mirror can be mounted half the lens spacing beyond any lens in the line from the second lens or more down the line. In order to use the first two lenses and tube as a matched source for testing the rest of the line, this second mirror, with a transmission of a fraction of one percent, was made to have optically flat surfaces with the front reflecting and back transmitting surfaces parallel to a small fraction of one light wavelength. This was necessary to prevent refraction of the beam from any inadvertent prism effects at this point.

Such a laser cavity should oscillate with the second mirror located half the lens spacing beyond any lens after the second one as long as the total line loss does not exceed the gain of the tube. After aligning the first two lenses, the mirrors and the tube for oscillation, further sections of the line were aligned, and the mirror moved down a few lenses at a time. Each time the mirror was moved the alignment and lens adjustments were trimmed for oscillation. This became more difficult as the line length increased and there were more adjustments to be optimized. If the system did not oscillate, there was nothing to indicate whether a change in any adjustment was better or worse. A coupled cavity technique was used to improve this situation. The partially transmitting mirror after the second lens was left in place so that the two-lens cavity formed an operating laser source. A third mirror was added at the end of the line away from this laser source. This formed a second cavity consisting of the transmission line without any active laser gain which was coupled to the laser section by the laser end mirror transmission. When the second cavity was brought into resonance by adjusting its far end mirror, there was an increase in the light output through the mirror at the other end of the laser source which could be read on a photocell meter. This reading increased when the long line adjustments were changed in the direction which reduced line loss and therefore improved the cavity Q. After adjusting for highest output from the laser end, the mirror between the active and inactive line sections was removed, and usually with, sometimes without, a slight trimming of the end mirror tilt adjustments, the long transmission line would oscillate as a single cavity.

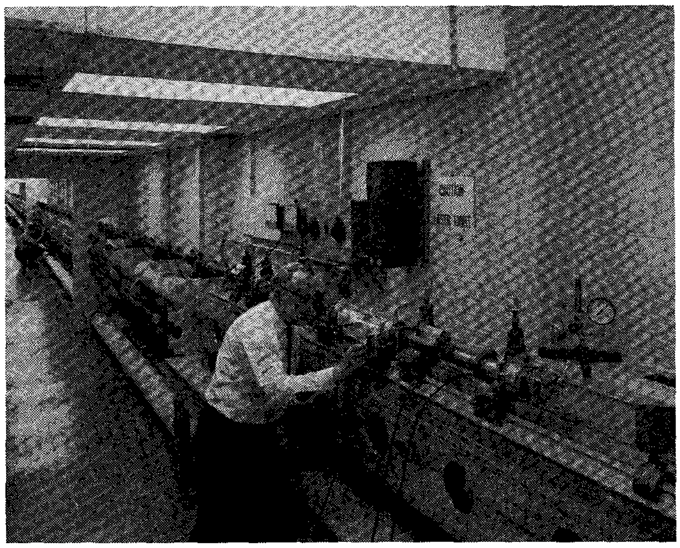

Fig. 1. Seventy-eight meter laser with a 30 cm helium-neon gas discharge tube and a transmission-line cavity consisting of 78 gas lenses in tandem.

Using this procedure, the transmission line was made to operate as a single lasing cavity for a total length of 78 meters between mirrors, using 78 lenses inside the cavity and a single 30 cm laser tube. The available gain of this tube after allowing for window and mirror losses is in the order of a little over one percent at the most, so the loss of the 78-lens line must be no more than this for the system to oscillate. This length was limited by the available corridor length, and not by line loss, so probably a longer line could be made to operate as a single cavity laser under these conditions. Adjustments of air flow and lens temperature were quite critical, indicating that tolerances on lens alignment are quite tight in order to obtain minimum losses.

The optical transmission-line installation is the top pipe line in Fig. 1 which shows the end where the laser tube is mounted and some of the 78 lenses. The mirror coupling the two sections in its adjustable mounting is partly visible just behind the author's head. There is a gas lens under each of the control boxes on the line.

This 78-meter long experimental gas lens transmission line has shown low loss as evidenced by its operation in a single laser cavity with only a small amount of gain to offset its loss. Further work to investigate its characteristics and performance is being done.

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